

Directions: Read each problem carefully. Number each problem. Show all work leading to an answer. Answer all parts of each question. Clearly label your answers. No Calculators!

1. (8 points) Evaluate the line integral $\int_C (xy + y^2 + z)ds$ over the curve C : the line segment from $(0,0,2)$ to $(2,3,1)$.
2. (20 points) Let $\vec{F} = xy\vec{i} + x^2y^3\vec{j}$ and let C be the triangle with vertices $(0,0)$, $(1,0)$, and $(1,2)$.
 - a. Directly compute the counterclockwise circulation of $\vec{F}$ around C .
 - b. Verify the answer from part a. by using Green's Theorem to compute the circulation of $\vec{F}$ around C .
3. (20 points) Let $\vec{F} = -x^2y\vec{i} + xy^2\vec{j}$ and let C be the circle $x^2 + y^2 = 4$.
 - a. Directly compute the flux of $\vec{F}$ across C .
 - b. Verify the answer from part a. by using Green's Theorem to verify the flux of $\vec{F}$ across C .
4. (16 points) Let $\vec{F} = 2xy\vec{i} + (x^2 + 2yz)\vec{j} + y^2\vec{k}$.
 - a. Determine if $\vec{F}$ is conservative on R^3 .
 - b. Find the potential function f of $\vec{F}$.
 - c. Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where C parametrized by $\vec{r}(t) = (4 \sin t)\vec{i} + (\cos t)\vec{j} + 3t\vec{k}$, for $0 \leq t \leq \frac{\pi}{3}$.
5. (16 points) Consider the surface S that is the cap of the paraboloid $z = 6 - x^2 - y^2$ above $z = 2$.
 - a. Find a parametrization of the surface. Include the parameter domain.
 - b. Find the surface area of S .
 - c. Evaluate $\iint_S \sqrt{1 + 4x^2 + 4y^2} d\sigma$.
6. (10 points) Let $\vec{F} = (y - z)\vec{i} + (z - x)\vec{j} + (x + z)\vec{k}$ and let C be the boundary of the triangle cut from the plane $2x + 3y + z = 6$ by the coordinate planes. Use an appropriate method to compute the circulation of $\vec{F}$ around C .
7. (10 points) Let $\vec{F} = (x^2 + yz)\vec{i} + (x^3 - 2xy - 2y^2)\vec{j} + (4yz + 1)\vec{k}$ and let S be the boundary of the region between the hyperbolic paraboloid $z = y^2 - x^2$, the cylinder $x^2 + y^2 = 9$, and the plane $z = 10$. Use an appropriate method to compute the outward flux of $\vec{F}$ across S .